Toughness averaging: Can we perform toughness upscaling with confidence?

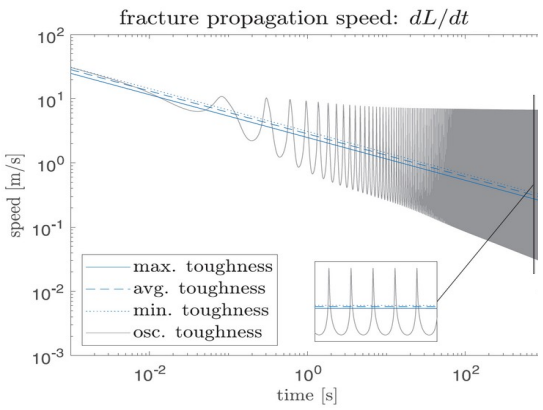
Gennady Mishuris

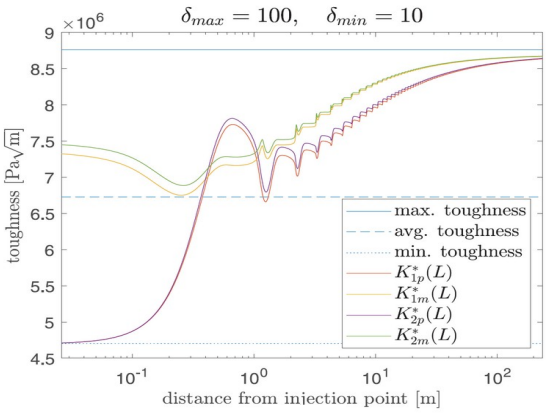
Thursday, January 12, 2023, 9 a.m. Central Time

Gennady Mishuris is a Professor of Mathematical Modelling at Aberystwyth University, Wales, UK. This work includes contributions from Gaspare Da Fies, Daniel Peck, and Martin Dutko. His presentation will be at 9:00 Central Time on Thursday, January 12, 2023. The topic is **"Toughness averaging: Can we perform toughness upscaling with confidence?"**

Abstract

This talk will discuss whether and how an averaging-based approach to the material toughness can be confidently utilized, even though toughness homogenisation has been proven to be an ill-defined concept in the framework of LEFM. Usually, various upscaling procedures are applied to achieve the goal. Recently, we have proposed an averaging-based approach that is dependent not only on the material but also process dependent parameters. Their definition comes from temporal averaging (in contrast to the spatial one). As a result, all introduced measures rely also on the instantaneous crack tip velocity during each specific process. The temporal average approach is general in nature (not specific to HF) and can be used in analysis of any stable fracture propagation process.





Numerous simulations have been performed utilizing our extremely accurate and effective in-house built time-space adaptive solver, which can obtain solutions for any of the 1D HF models (PKN, KGD, Radial, 3PD, non-local PKN) with arbitrary (fixed) fluid rheology, leak off law and pumping regime. The solver uses the crack opening and the fluid velocity as the basic unknowns in contrast to the

conventional crack opening and fluid pressure pair [1]. We analyse the KGD and Radial HF models in an elastic homogeneous material characterised by periodic toughness distributions (see in figure (a), one of possible instantaneous crack velocity profiles). The simulations allow us to demonstrate the temporal-averaging concept [2], showing, among others, how the effective (averaged) toughness approaches its maximum value when the crack is sufficiently long (Figure (b)), as was elegantly claimed by Dontsov and Suarez-Rivera (2021).

We discuss various peculiarities of the HF propagation in such a media. In particular, we show how local energy redistribution affects the process, resulting in local (in time and space) changes of the propagation regime. For example, even if both the maximum and minimum values of the toughness distribution correspond solely to the high toughness regime (under a given fluid rate), local regions exhibiting small toughness/viscosity dominated behaviour are apparent. The reverse also holds true, if both extremal values relate to the small toughness regime, then regions displaying behaviour associated with high toughness can be locally observed, being more dominant for a long crack. Another interesting feature of the measures: even though the toughness and energy release rate fracture criteria are equivalent in the problem under consideration (homogeneous elastic material), temporal averaging based on the energy argument appears to be more accurate. Finally, we discuss possible applications of the introduced measures.

References

[1] Wrobel, M. Mishuris, G. (2015) Hydraulic fracture revisited: Particle velocity-based simulation. IJES, 94, 23-58, 10.1016/j.ijengsci.2015.04.003
[2] Peck, D., Da Fies, G., Dutko, M., Mishuris, G. (2022). A temporal averaging-based approach to toughness homogenisation in heterogeneous material. Mathematics and Mechanics of Solids, https://doi.org/10.1177/10812865221117.

Biography

Professor Mishuris holds a personal chair in Mathematical Modelling in the Department of Mathematics at Aberystwyth University since 2007, when the Wales Institute of Mathematical and Computational Sciences (WIMCS) was created. He graduated in 1982 from Leningrad (now Saint Petersburg) State University where he also completed his PhD thesis in 1985. He worked in the USSR (Russia), before moving to Poland in 1993 where he defended his habilitation (DSc) degree in Cracow University of Technology in 1999. Professor Mishuris was awarded the Alexander von Humboldt Fellowship to do research in Germany (Erlangen-Nuremberg University 2000-2001) and the Leverhulme Trust Visiting Fellowship in the UK, 2004.

His resent research interests lie in the various directions of applied mathematics and applications: matrix factorisation and Wiener-Hopf technique; modelling of dynamic fractures in discrete structures, hydrofracturing, biomechanics, thin nonlinear interphases and others. Professor Mishuris is a Fellow of the Learned Society of Wales (Wales Academy of Sciences) and the Royal Society Wolfson Research Merit Award holder.