Discrete Element Based Hydraulic Fracture Model

Test Case 3: Single fracture in homogenous poroelastic, thermoelastic media
(a) Newtonian fluid without proppant in a poroelastic media

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Discrete Element Method

1. Discrete element tracking (Yade): Newton’s 2\textsuperscript{nd} Law: $\ddot{u} = F/m$

2. Determination of forces (particle interactions):

\[ k_n = \frac{E_1 r_1 E_2 r_2}{E_1 r + E_2 r_2} \quad \& \quad k_s = v k_n \big| F_n = k_n \Delta D \quad \& \quad F_s = F_{s,prev} + k_s \Delta u_s \]

3. Failure criteria (Scholtes and Donze 2012):

\[ F_{n,\text{max}} = -t A_{\text{int}} \quad \& \quad F_{s,\text{max}} = F_n \tan \phi + c A_{\text{int}} \]

4. Fluid coupling (Yade, Chareyre et al. 2012):

\[ [G][P] = [E][\dot{X}] + Q_q \]

5. Triangulation created using particles as nodes

6. Conductance governed by Poiseuille’s law (Papachristos, 2017):

- non fractured, $k = \alpha \frac{A_{ij} R_{ij}^h}{\mu}$
- fractured, $k = \frac{h^3}{12 \mu}$

7. Pressure and viscous forces on particles:

\[ F_p = A_p (p_i - p_j) n \quad \& \quad F_{v,\text{total}} = A_f (p_i - p_j) n \]

\[ F_{v,p} = F_{v,\text{total}} \gamma \quad \& \quad \gamma = \frac{A_p}{A_{\text{total}}} \]

$A_f=$pore throat cross section, $p =$ pore pressure, $G =$ conductance matrix, $E\dot{X} =$ rate of volume change, $P =$ pore pressures, $Q_q =$ source term, $F =$ force, $m =$ mass, $\ddot{u} =$ acceleration, $\mu =$ dynamic viscosity, $v =$ microscopic Poisson’s ratio, $k =$ stiffness, $t =$ tensile strength, $A_{\text{int}} =$ interaction area, $c =$ cohesion, $k =$ permeability, $R_{ij}^h =$ hydraulic radius, $h =$ separation distance, $\Delta D =$ particle overlap, $A_p =$ area of particle on pore, $F_{v/p}$ = viscous/pressure force, $\phi =$ friction angle, $\alpha =$ perm. coeff.
Numerical Methods and Assumptions

- Particle position - explicit finite difference
- Fluid flow – pore finite volume

Model Assumptions:
- Matrix permeability – Poiseuille’s law
- Fracture permeability – parallel plate approximation
- Mohr-coulomb failure criteria based on particle size
- Broken bonds contain residual fracture width
- Calibrated micro-parameters yield emergent behaviors according to specified macro parameters
- 10 cm perforation depth
- Constant pressure and stress boundary conditions
- No vertical flow out of layer of interest

<table>
<thead>
<tr>
<th>Micro parameter</th>
<th>Value (DEM)</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E_i )</td>
<td>32 (GPa)</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>( k_s/k_n )</td>
<td>0.05</td>
<td>Stiffness ratio</td>
</tr>
<tr>
<td>( \phi )</td>
<td>25°</td>
<td>friction angle</td>
</tr>
<tr>
<td>( c )</td>
<td>15 MPa</td>
<td>cohesion</td>
</tr>
<tr>
<td>( t )</td>
<td>2.3 MPa</td>
<td>tensile strength</td>
</tr>
<tr>
<td>( \gamma_{int} )</td>
<td>1.329</td>
<td>interaction range</td>
</tr>
<tr>
<td>( r )</td>
<td>1.5 m ± 0.25</td>
<td>particle radii</td>
</tr>
<tr>
<td>( \rho )</td>
<td>5000 kg/m³</td>
<td>particle density</td>
</tr>
<tr>
<td>( n )</td>
<td>0.38</td>
<td>pack porosity</td>
</tr>
<tr>
<td>( P_p )</td>
<td>27 MPa</td>
<td>reservoir pressure</td>
</tr>
<tr>
<td>( k_{factor} )</td>
<td>9e-16</td>
<td>permeability factor</td>
</tr>
<tr>
<td>( K_{fluid} )</td>
<td>2.2 GPa</td>
<td>fluid bulk modulus</td>
</tr>
<tr>
<td>( \mu )</td>
<td>0.001 Pa*s</td>
<td>viscosity</td>
</tr>
<tr>
<td>( h_{residual} )</td>
<td>1e-6 m</td>
<td>residual aperture</td>
</tr>
</tbody>
</table>
Fracture Width

Distance between non-cohesive particles

\[ b_{avg} = \frac{\sum_{i=1}^{n} d_{0i} - (d_{1i} + d_{2i})}{n} \]

- \( b_{avg} \) = average width
- \( d_{0i} \) = distance between particle 1 and particle 2 centers
- \( n \) = number of noncohesive interactions
- \( d_{1i} \) = particle 1 radius
Leak-off Rate

Total flux through fracture face

\[ q = \sum_{i=1}^{n} k_i (p_{frac_i} - p_{matrix_i}) \]

- \( p_{frac_i} \) = pressure in fractured pore abutting fracture face
- \( p_{matrix_i} \) = pressure in neighbor matrix pore
- \( n \) = number of fractured pores on fracture face
- \( k_i \) = conductivity factor computed using Poiseuille’s law
*Fractured cells shown in green*
Extra Plots – analytical comparison

Width of pressurized penny shaped crack (Sneddon and Elliot 1946)

\[ w(r) = \frac{8p_{\text{net}}R}{\pi E'} \sqrt{1 - \left(\frac{r}{R}\right)^2} \]


\[ \Delta V_f = q_i \Delta t_p - V_L \]
\[ V_L = 2C_L A_L \sqrt{t} + S_p \]

\[ p_{\text{net},c} = \left(\frac{2\pi^3 \gamma_F E'^2}{3V}\right)^{1/5} \]

\[ V_L = \text{leak off volume}, \ q_i = \text{injection flow rate} \]

\( w = \text{fracture width}, \ p_{\text{net}} = \text{net pressure}, \ C_L = \text{leak off coeff} = 1e-5 \ \text{ft/\sqrt{min}} \]
\( E' = \text{Young's modulus}, \ \gamma_F = \text{fracture energy}, \ R = \text{fracture radius} \)