**Abstract**

This paper presents an enhanced pseudo-3D (P3D) model for hydraulic fracturing (HF). This model substantially increases accuracy of the classical P3D model, while retaining its computational efficiency. As a result, this model features a unique combination of accuracy and computational efficiency. In order to raise the accuracy, the weakest points of the classical P3D model are addressed. Firstly, a correction for the viscous length growth is introduced, which incorporates the effects of fluid viscosity on height growth. Secondly, the local elasticity equation is replaced with its non-local counterpart. Such a change permitted to capture the transition from the KGD-like behavior near the fracture tip to the local elasticity near the fracture center. In addition, this modification allowed to incorporate the effect of lateral fracture toughness through the universal tip asymptotic solution. Finally, a curved fracture tip is introduced, which also helps to predict radial fracture shape at early times. The computational efficiency of the model remains high since the numerical formulation requires solution of a one-dimensional problem, as opposed to the two-dimensional problem for a full planar model.

**Model description**

Model features and assumptions:
- Three stress and toughness layers.
- Homogeneous elastic properties.
- Fluid pressure is uniform in each vertical cross-section.
- Plane strain analysis is used for fracture height and width in each vertical cross-section.
- Radial solution in the tip region.
- Non-local elasticity.
- Universal tip asymptotic solution for fracture propagation.
- Viscous viscosity height growth.
- No leak-off.

**Governing equations**

- Effective width:
  \[ w(x) = \frac{1}{H} \int_{-H/2}^{H/2} \sqrt{w(x, z) \, dz} \]  

- Plane strain solution in each vertical cross-section:
  \[ K(x, z), \quad w(x, z) \]  

See e.g. [3] for the exact expressions of these functions.

**Fracture opening solutions**

- Stage 1: growth within the main layer.
- Stage 2: widening within the main layer.
- Stage 3: growth through one barrier.
- Stage 4: growth through both barriers.

**Model validation**

**Symmetric stress barriers, uniform toughness**

- ILSA: \( K^* = 0 \)
- EPID: \( K^* = 0 \)
- EPID: \( K^* = 3.0 \text{ MPa·m}^{1/2} \)
- Experiment: \( K^* = 0 \)

**"Extreme" case**

**References**